SURVEYS COMBINING PROBABILITY AND QUOTA METHODS OF SAMPLING

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INTRODUCTION

This paper discusses a Bayesian analysis of a model in which it is possible to have *full probability and quota sampling* within the same set of primary sampling units for the same survey.

The study was inspired by the analysis of biased measurement in the textbook of Pratt, Raiffa and Schlaifer (1965), but in order to better approximate the sampling procedures actually employed in survey research, that simple model has been considerably elaborated upon, resulting in optimization formulas that are more complicated.

We consider the following specification:

$$\tilde{x}_{uij} = \mu_u + \tilde{\gamma}_i + \tilde{\varepsilon}_{uij} \qquad (2.1)$$

$$\tilde{x}_{bik} = \mu_b + \tilde{\gamma}_i + \tilde{\varepsilon}_{bik}$$

$$j = 1,2,\ldots,n_u$$
$$k = 1,2,\ldots,n_b$$
$$i = 1,2,\ldots,p$$

$$E(\tilde{\gamma}_i) = E(\tilde{\varepsilon}_{uij}) = E(\tilde{\varepsilon}_{bik}) = 0 \qquad (2.2)$$

$$E(\tilde{\gamma}_i\tilde{\varepsilon}_{uij}) = E(\tilde{\gamma}_i\tilde{\varepsilon}_{bik}) = E(\tilde{\varepsilon}_{uij}\tilde{\varepsilon}_{bik}) = 0 \qquad (2.3)$$

$\tilde{x}_{uij}$ and $\tilde{x}_{bik}$ are observations from unbiased (probability) and biased (quota) measurement processes, respectively.

$\tilde{\gamma}_i$ is a random effect that is held in common by all observations, biased or not, in the i^{th} PSU.

$\tilde{\varepsilon}_{uij}$ is a second-stage disturbance term unique to the j^{th} observation in the full probability sample within the i^{th} PSU.

$\tilde{\varepsilon}_{bik}$ is correspondingly a second-stage random disturbance in quota sampling.

μ_u and μ_b are respectively the expected values of the $\tilde{x}_{uij}$ and $\tilde{x}_{bik}$, and $\beta = \mu_u - \mu_b$ is the bias.

Define

$$v_u = E(\tilde{\varepsilon}^2_{uij})$$
$$v_b = E(\tilde{\varepsilon}^2_{bik}) \qquad (2.4)$$
$$v_c = E(\tilde{\gamma}^2_i),$$

the variances of the random components on the right hand side of (2.1). It follows that

$$V(\tilde{x}_{uij}) = v_c + v_u$$
$$V(\tilde{x}_{bik}) = v_c + v_b \qquad (2.5)$$

and $\mathrm{Cov}(\tilde{x}_{uij}, \tilde{x}_{bik}) = v_c$.

Summarizing, our model assumes that p PSUs are randomly selected and that either full probability or quota sampling *or both methods* are applied to the units within each PSU. The within-PSU sample sizes n_u and n_b are constant across PSUs. Furthermore, it is assumed that both probability and quota observations are subject to the same random PSU effect; i.e., the bias conditional on PSU i is equal to the unconditional bias, $\mu_b - \mu_u$. The latter assumption implies that if, for example, a certain PSU has mean income that is higher than the national average, μ_u, the same random effect is present among the responses in quota sampling, although the within PSU average incomes for probability and quota sampling may differ substantially because of the fundamental bias, $\mu_b - \mu_u$.

Finally, we realize that actual survey practice involves sampling from finite populations, but for simplicity, we treat the selected PSUs as independent. Further, we assume that the within PSU observations in the samples of sizes n_u and n_b are independent conditional on γ_i.

PRIOR-TO-POSTERIOR ANALYSIS

Assume that the prior distribution of $(\mu_u, \mu_b)^t$ is bivariate normal with mean and variance

$$E\begin{bmatrix}\tilde{\mu}_u\\ \tilde{\mu}_b\end{bmatrix} = \begin{bmatrix}\bar{x}'_u\\ \bar{x}'_b\end{bmatrix},$$

$$V\begin{bmatrix}\tilde{\mu}_u\\ \mu_b\end{bmatrix} = \begin{bmatrix}V'_{uu} & V'_{ub}\\ V'_{ub} & V'_{bb}\end{bmatrix}, \qquad (3.1)$$

where V'_{ub} is the prior covariance of $\tilde{\mu}_u$ and $\tilde{\mu}_b$. Define

$$\tilde{\bar{x}}_u = \sum_{i=1}^{p}\sum_{j=1}^{n_u} \tilde{x}_{uij}/pn_u, \qquad (3.2)$$

the arithmetic mean of the unbiased observations, and

$$\tilde{\bar{x}}_b = \sum_{i=1}^{p}\sum_{k=1}^{n_b} \tilde{x}_{bik}/pn_b, \qquad (3.3)$$

the arithmetic mean of the quota sample.

Then the mean and variance of $(\tilde{\bar{x}}_u, \tilde{\bar{x}}_b)^t$, conditional on $(\mu_u, \mu_b)^t$ is

$$E\left[\begin{matrix}\tilde{\bar{x}}_u\\ \tilde{\bar{x}}_b\end{matrix}\middle|\begin{matrix}\mu_u\\ \mu_b\end{matrix}\right] = \begin{bmatrix}\mu_u\\ \mu_b\end{bmatrix},$$

$$V\left[\begin{matrix}\tilde{\bar{x}}_u\\ \tilde{\bar{x}}_b\end{matrix}\middle|\begin{matrix}\mu_u\\ \mu_b\end{matrix}\right] = \begin{bmatrix}v_c/p + v_u/pn_u & v_c/p\\ v_c/p & v_c/p+v_b/pn_b\end{bmatrix}. \qquad (3.4)$$

If we assume that the $\tilde{\gamma}_i$, $\tilde{\varepsilon}_{uij}$, and $\tilde{\varepsilon}_{bik}$ are normally distributed with p, n_u, and n_b, all > 0, then $(\tilde{\bar{x}}_u, \tilde{\bar{x}}_b)^t$ is conditionally bivariate normal. With nonnormal random components, the central limit theorem can be invoked to treat the sample means as approximately normal and approximately sufficient for $(\mu_u, \mu_b)^t$.

Define

$$H' = \begin{bmatrix} H'_{uu} & H'_{ub} \\ H'_{ub} & H'_{bb} \end{bmatrix} = \begin{bmatrix} V'_{uu} & V'_{ub} \\ V'_{ub} & V'_{bb} \end{bmatrix}^{-1}, \quad (3.5)$$

$$H = (n_u v_c v_b + n_b v_c v_u + v_u v_b)^{-1} \begin{bmatrix} pn_u(n_b v_c + v_b) & -pn_u n_b v_c \\ -pn_u n_b v_c & pn_b(n_u v_c + v_u) \end{bmatrix}, \quad (3.6)$$

and

$$H'' = H' + H. \quad (3.7)$$

It follows from the standard Bayesian development for the bivariate normal case that $\tilde{\mu}_u$ and $\tilde{\mu}_b$ have a joint posterior distribution that is normal, with mean and variance

$$\begin{bmatrix} \bar{x}''_u \\ \bar{x}''_b \end{bmatrix} = H''^{-1} \left[H' \begin{bmatrix} \bar{x}'_u \\ \bar{x}'_b \end{bmatrix} + H \begin{bmatrix} \bar{x}_u \\ \bar{x}_b \end{bmatrix} \right],$$

$$V'' = \begin{bmatrix} V''_{uu} & V''_{ub} \\ V''_{ub} & V''_{bb} \end{bmatrix} = H''^{-1}. \quad (3.8)$$

The posterior expectation of $\tilde{\mu}_u$ is of particular interest to us, and straightforward algebra yields

$$\bar{x}''_u = \bar{x}'_u + w_u(\bar{x}_u - \bar{x}'_u) + w_b(\bar{x}_b - \bar{x}'_b), \quad (3.9)$$

where

$$w_u = (n_u v_c v_b + n_b v_c v_u + v_u v_b)^{-1} [V''_{uu} pn_u(n_b v_c + v_b) - V''_{ub} pn_u n_b v_c]$$

and (3.10)

$$w_b = (n_u v_c v_b + n_b v_c v_u + v_u v_b)^{-1} [V''_{ub} pn_b(n_u v_c + v_u) - V''_{uu} pn_u n_b v_c].$$

Thus, depending on the posterior covariance structure, the posterior expectation of the mean of the unbiased process is affected by sample deviations of both the probability and quota means from their respective prior expectations.

In subsequent developments we shall require the posterior variance of $\tilde{\mu}_u$,

$$V''_{uu} = (H''_{uu} - H''^2_{ub} H''^{-1}_{bb})^{-1}. \quad (3.11)$$

It will be more convenient to work with the posterior precision, which, substituting from (3.5) and (3.6), can be written in the form

$$V''^{-1}_{uu} = u/v, \quad (3.12)$$

where

$$u = (H'_{uu}H'_{bb} - H'^2_{ub})(n_u v_c v_b + n_b v_c v_u + v_u v_b) + p^2 n_u n_b + H'_{uu} pn_b(v_u + n_u v_c) \quad (3.13)$$
$$+ H'_{bb} pn_u(v_b + n_b v_c) + 2 H'_{ub} pn_u n_b v_c,$$

and

$$v = H'_{bb}(n_u v_c v_b + n_b v_c v_u + v_u v_b) \quad (3.14)$$
$$+ pn_u n_b v_c + pn_b v_u$$

It can be shown that if n_b is zero, implying a design that is totally devoted to unbiased sample selection, expressions (3.9) through (3.14) collapse to the usual univariate normal result.

OPTIMUM ALLOCATION OF A FIXED BUDGET

Consider a survey in which the principal task is point estimation of $\tilde{\mu}_u$, but for which the economic resources available are a fixed dollar amount k*. (This is not an uncommon situation in actual practice.)

Defining

k_c = the cost of "setting up" a staff for this survey in a single PSU,

k_u = the per unit cost of a within-PSU probability observation,

and k_b = the per unit cost of a quota observation, (4.1)

we write

$$k^* = k_c p + k_u pn_u + k_b pn_b. \quad (4.2)$$

It is assumed that elements of fixed cost for planning, overall supervision, etc., have been subtracted out of k*.

With a quadratic loss function of the error in estimating $\tilde{\mu}_u$, the optimum Bayesian estimator is $\bar{x}''_u$, the posterior mean. The posterior expected loss is proportional to the posterior variance of $\tilde{\mu}_u$. We shall consider the case where v_c, v_u and v_b are known, and $\bar{x}''_u$ will be used to estimate $\tilde{\mu}_u$. Given the fixed amount k*, we wish to allocate it between probability and quota sample observations in such a way as to maximize the preposterior variance of $\tilde{\bar{x}}''_u$, the controllable factor in the expected net gain from sampling (ENGS). Since

$$V(\tilde{\bar{x}}''_u) = V'_{uu} - V''_{uu}, \quad (4.3)$$

we must choose p, the number of PSUs, n_u and n_b in order to minimize V''_{uu}, given in (3.11) above, within the fixed budget k*.

Using (4.2) to express n_b in terms of p and n_u, we substitute in (3.13) and (3.14) and rearrange to get

$$\begin{aligned} u = {} & pv_u(|H'|v_b k_b - |H'|v_c k_c + H'_{uu}k^*) \\ & - p^2 H'_{uu} v_u k_c + |H'| v_u v_c k^* \quad (4.4) \\ & + n_u [p|H'|v_c(v_b k_b - v_u k_u) \\ & + pv_c k^*(H'_{uu} + H'_{bb} + 2H'_{ub}) \\ & + p^2(H'_{bb} v_b k_b - H'_{uu} v_u k_u + k^*) \\ & - p^2 v_c k_c (H'_{uu} + H'_{bb} + 2H'_{ub}) - p^3 k_c] \\ & - n_u^2 [p^2 k_u v_c (H'_{uu} + H'_{bb} + 2H'_{ub}) + p^3 k_u], \end{aligned}$$

and

$$\begin{aligned} v = {} & pv_u [H'_{bb}(v_b k_b - v_c k_c) + k^*] \\ & - p^2 v_u k_c + H'_{bb} v_u v_c k^* \\ & + n_u [pv_c (H'_{bb} [v_b k_b - v_u k_u] + k^*) \end{aligned}$$

$$- p^2(v_c k_c + v_u k_u)] - n_u^2 p^2 v_c k_u. \qquad (4.5)$$

The desired approach is to maximize the posterior precision (3.12) by differentiating with respect to p and n_u, and setting the partial derivatives equal to zero. The resulting equations are very complicated, however, and a simultaneous solution for n_u and p will require numerical methods of analysis. In the following, we shall concentrate on the optimal choice of n_u for a fixed value of p, the number of PSUs, and then explore through examples, the effect of changes in p. We shall be especially interested in a p of about 100 because in typical surveys, as alluded to above, this may be a practical maximum that an organization can support in ongoing sampling operations. From the other side, there may be nonstatistical pressure to have at least that number of PSUs in order to satisfy demands for information from regional interest groups. In certain other survey situations, there may be reasons for keeping p small. For example, in the evaluation of a government poverty program with serious political implications, it may be desirable to keep the study down to 5 or 10 metropolitan areas for reasons outside of the statistician's control.

Differentiating (3.12) with respect to n_u, and using (4.4) and (4.5) to define u and v, we set

$$vdu - udv = 0 \qquad (4.6)$$

The result is a quadratic equation in n_u,

$$an_u^2 + bn_u + c = 0 \qquad (4.7)$$

where

$$a = p^4 + 2p^3 v_c(H'_{bb} + H'_{ub}) + p^2 v_c^2 (H'_{bb}+H'_{ub})^2 (1- [v_b/v_u]q_b), \qquad (4.8)$$

$$b = 2p^4 q_c + 2p^3 [2v_c q_c (H'_{bb}+H'_{ub}) - v_b q_b H'_{bb} - q^*] + 2p^2 v_c [(v_c q_c - v_b q_b)(H'_{bb}+H'_{ub})^2 - 2q^*(H'_{bb}+H'_{ub})] - 2pv_c^2 q^*(H'_{bb}+H'_{ub})^2, \qquad (4.9)$$

and

$$c = p^4 q_c^2 + 2p^3 q_c [v_c q_c (H'_{bb}+H'_{ub}) - v_b q_b H'_{bb} - q^*] \qquad (4.10)$$
$$+ p^2 [v_b q_b (2q^* H'_{bb} - v_u H'^2_{ub}) + q^{*2} - 4v_c q_c q^*(H'_{bb}+H'_{ub}) + (v_c q_c [H'_{bb}+H'_{ub}] - v_b q_b H'_{bb})^2]$$
$$+ 2pv_c q^* [(H'_{bb}+H'_{ub})(q^*+v_b q_b H'_{bb}) - v_c q_c (H'_{bb}+H'_{ub})^2] + v_c^2 q^{*2} (H'_{bb}+H'_{ub})^2.$$

In (4.8), (4.9) and (4.10) the costs are expressed relative to k_u, the per unit cost of an unbiased observation:

$$q_b = k_b/k_u, \quad q_c = k_c/k_u, \quad \text{and} \quad q^* = k^*/k_u. \qquad (4.11)$$

In analyzing the effect of prior parameters on the optimal allocation between full probability and quota sampling for a fixed value of p, there is no loss in generality from assuming that $q_c = 0$. Thus, (4.9) and (4.10) are simplified to

$$b = -2p^3 (v_b q_b H'_{bb} + q^*) - 2p^2 v_c (H'_{bb}+H'_{ub}) [v_b q_b (H'_{bb}+H'_{ub}) + q^*] - 2pv_c^2 q^* (H'_{bb}+H'_{ub})^2, \qquad (4.12)$$

and

$$c = p^2 v_b q_b (2q^* H'_{bb} - v_u H'^2_{ub} + v_b q_b H'^2_{bb}) + 2pv_c q^* (H'_{bb}+H'_{ub})(q^*+v_b q_b H'_{bb}) + v_c^2 q^{*2} (H'_{bb}+H'_{ub})^2. \qquad (4.13)$$

Using (4.8), (4.12) and (4.13), it can be shown through considerable algebraic manipulation that $b^2 - 4ac$ is a perfect square, x^2, such that

$$x = 2p(v_b/v_u)^{1/2} q_b^{1/2} [p^2 v_u H'_{ub} + pv_c (H'_{bb}+H'_{ub})(v_b q_b H'_{bb} + v_u H'_{ub} + q^*) + v_c q^* (H'_{bb}+H'_{ub})^2]. \qquad (4.14)$$

Thus the two roots of (4.7), when $q_c = 0$, are given by

$$n_u = (-b \pm x)/2a, \text{ or} \qquad (4.15)$$

$$n_u = [p^2 (v_b q_b H'_{bb} + q^* \pm v_b^{1/2} q_b^{1/2} v_u^{1/2} H'_{ub}) + pv_c (H'_{bb}+H'_{ub}) \{v_b q_b [(1 \pm v_b^{1/2} q_b^{1/2} v_u^{-1/2}) H'_{bb} + (1 \pm v_u^{1/2} v_b^{-1/2} q_b^{-1/2}) H'_{ub}] + (2 \pm v_b^{1/2} q_b^{1/2} v_u^{-1/2}) q^*\} + v_c^2 q^* (1 \pm v_b^{1/2} q_b^{1/2} v_u^{-1/2})(H'_{bb}+H'_{ub})^2]/a^*, \qquad (4.16)$$

where

$$a^* = p^3 + 2p^2 v_c (H'_{bb}+H'_{ub}) + pv_c^2 (H'_{bb}+H'_{ub})^2 [1-(v_b/v_u) q_b]. \qquad (4.17)$$

We shall call a solution to (4.15) "feasible" if it falls in the range $[0, q^*/p]$, where q^*/p is the maximum allowed by the budget constraint. When one root is in the feasible range the other is either negative or exceeds q^*/p, and the precision is maximized when n equals the feasible n_u. If both roots are greater than q^*/p, the optimal sample allocation is $n_u = q^*/p$. If one root is negative and the other greater than q^*/p, the best allocation is $n_u = 0$. The number of observations to be selected by quota methods in each PSU is from (4.2) and (4.11)

$$n_b = (q^* - n_u p)/(pq_b). \qquad (4.18)$$

SENSITIVITY ANALYSIS

The complexity of expressions (4.16) and (4.17) makes any formal analysis of the effects of prior parameters on n_u a near impossible task. We shall, however, examine some interesting special cases, and consider a few examples:

a. The case of $p = 1$, $v_c = 0$.

This is the model of Pratt, Raiffa and Schlaifer.

From (4.16) and (4.17) we have

$$n_u = q^* + v_b q_b (H'_{bb} \pm v_u^{\frac{1}{2}} v_b^{-\frac{1}{2}} q_b^{-\frac{1}{2}} H'_{ub}). \quad (6.1)$$

We must consider the following possible situations:

(1) $H'_{bb} > v_u^{\frac{1}{2}} v_b^{-\frac{1}{2}} q_b^{-\frac{1}{2}} |H'_{ub}|$, in which case both roots of (4.7) are greater than q^*, the total budget. The optimal allocation is to put all resources into full probability sampling, $n_u = q^*$, $n_b = 0$.

(2) $v_b q_b (v_u^{\frac{1}{2}} v_b^{-\frac{1}{2}} q_b^{-\frac{1}{2}} |H'_{ub}| - H'_{bb}) > q^*$, in which case one root of (4.7) is greater than q^* and the other is negative. The optimal allocation is to use the total budget for quota sampling;

$$n_b = q^*/q_b, \quad n_u = 0.$$

(3) $0 < v_b q_b (v_u^{\frac{1}{2}} v_b^{-\frac{1}{2}} q_b^{-\frac{1}{2}} |H'_{ub}| - H'_{bb}) < q^*$. The optimal allocation is

$$n_u = q^* - v_b q_b (v_u^{\frac{1}{2}} v_b^{-\frac{1}{2}} q_b^{-\frac{1}{2}} |H'_{ub}| - H'_{bb})$$

and

$$n_b = v_b (v_u^{\frac{1}{2}} v_b^{-\frac{1}{2}} q_b^{-\frac{1}{2}} |H'_{ub}| - H'_{bb}). \quad (6.2)$$

b. The case of $\tilde{\mu}_u$ and $\tilde{\beta}$ independent a priori.

$\tilde{\mu}_u$ and $\tilde{\beta}$ uncorrelated implies that $V'_{ub} = V'_{uu}$. Then from substitution of $(H'_{bb} + H'_{ub}) = 0$ in (4.16),

$$n_u = [v_b q_b H'_{bb}(1 \pm v_u^{\frac{1}{2}} v_b^{-\frac{1}{2}} q_b^{-\frac{1}{2}}) + q^*]/p \quad (6.3)$$

Note that the PSU component of the process variance, v_c, has no effect on the sample allocation. (It does, however, have a marked effect on the posterior variance.) It is clear that

$$n_u = [v_b q_b H'_{bb}(1 - v_u^{\frac{1}{2}} v_b^{-\frac{1}{2}} q_b^{-\frac{1}{2}}) + q^*]/p \quad (6.4)$$

is the only feasible root within the budget constraint, and that $n_u < q^*/p$ requires

$$v_u^{\frac{1}{2}} v_b^{-\frac{1}{2}} q_b^{-\frac{1}{2}} > 1. \quad (6.5)$$

It is instructive to assume that $v_u = v_b$ and write (6.4) as

$$n_u = (q^*/p) + [V'_{uu} v_b q_b^{\frac{1}{2}} (q_b^{\frac{1}{2}} - 1)]/(\det V')p. \quad (6.6)$$

We see that n_u is only slightly decreased through q_b by a reduction in the relative cost of quota sampling, whereas a large v_b or high prior correlation between $\tilde{\mu}_u$ and $\tilde{\mu}_b$ can have more direct impact. With the assumption $v_u = v_b$, we can express v_b and V'_{bb} in terms of V'_{uu}:

$$v_b = c_1 V'_{uu}$$

$$V'_{bb} = c_2 V'_{uu}. \quad (6.7)$$

It follows from the case specification, $V'_{uu} = V'_{ub}$, that

$$\rho_{ub} = c_2^{-\frac{1}{2}}, \quad (6.8)$$

and from (6.6)

$$n_u = (q^*/p) + \rho_{ub}^2 c_1 q_b^{\frac{1}{2}} (q_b^{\frac{1}{2}} - 1)/(1 - \rho_{ub}^2)p. \quad (6.9)$$

Hence the optimal allocation is invariant given q^*, p, q_b, the ratio of $v_b = v_u$ to V'_{uu}, and the prior correlation, ρ_{ub}, between $\tilde{\mu}_u$ and $\tilde{\mu}_b$.

We also see that the optimal n_u, relative to its maximum possible value, is

$$1 + \rho_{ub}^2 c_1 q_b^{\frac{1}{2}} (q_b^{\frac{1}{2}} - 1)/(1 - \rho_{ub}^2) q^*, \quad (6.10)$$

which does not depend on p.

In order to examine this case further, in Table 2 we display the values of n_u that are optimal for various combinations of c_1 and ρ_{ub}, given

$$p = 100, \quad q_b = .1, \quad q^* = 1600,$$

$$V'_{uu} = V'_{ub} = 1. \quad (6.11)$$

$$\rho_c = v_c/(v_c + v_b) = 0.$$

The magnitudes of V'_{uu} and ρ_c do not affect the allocation given by (6.9) but they are necessary in order to evaluate the posterior variance. For the problem of estimating the proportion of a dichotomous population possessing one of the attributes $V'_{uu} = 1$ could well represent an approximate "vague" or diffuse prior for $\tilde{\mu}_u$ known to lie in [0, 1].

The $q^* = 1600$ is a typical total sample size for a household survey, spread over 100 PSUs.

Table 2 shows that in the example of the dichotomous population, where it is unlikely that $v_b = v_u$ would be much greater than .1, one must have a prior ρ_{ub} beyond the range of the table for any quota sampling to be desirable.

Table 3 shows the posterior precision relative to the optimum if the survey planner were to ignore Table 2 and devote all of his resources to quota sampling, with $n_b = 160$ ($q_b = .1$).

Consider the example of $c_1 = 0.1$ and $\rho_{ub} = 0.95$. $V'_{uu} = 1$ implies $v_b = v_u = 0.1$ and $V'_{bb} = 1.10803$. It can be shown in (3.11) - (3.14) that with the optimal allocation, $n_u = 16$, the posterior standard deviation of $\tilde{\mu}_u$ is

$$V''^{\frac{1}{2}}_{uu} = 0.0079.$$

The relative precision of full quota sampling, shown in Table 3 as 0.001 is, more exactly, equal to

0.000641.

2. VALUES OF n_u DETERMINED BY $c_1 = v_b/V'_{uu}$ AND ρ_{ub}.
($p=100$, $q_b=.1$, $q^*=1600$, $v_b=v_u$, $\rho_c=0$, $V'_{uu}=V'_{ub}=1$, i.e. $\rho_{u\beta}=0$.

c_1	ρ_{ub} .100	.300	.500	.700	.800	.900	.950	.990	.995	.999
.01	16	16	16	16	16	16	16	16	16	16
.1	16	16	16	16	16	16	16	16	16	16
.3	16	16	16	16	16	16	16	16	16	16
5	16	16	16	16	16	16	16	16	16	15
.7	16	16	16	16	16	16	16	16	16	15
1	16	16	16	16	16	16	16	16	16	15
10	16	16	16	16	16	16	16	15	14	5
25	16	16	16	16	16	16	15	13	11	0
100	16	16	16	16	16	15	14	5	0	0
1000	16	16	15	14	12	7	0	0	0	0

3. PRECISION OF TOTAL QUOTA SAMPLING RELATIVE TO OPTIMUM ALLOCATION SHOWN IN TABLE 2

c_1	ρ_{ub} .100	.300	.500	.700	.800	.900	.950	.990	.995	.999
.01	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003
.1	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.006	0.031
.3	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.009	0.019	0.093
.5	0.000	0.000	0.000	0.001	0.001	0.002	0.003	0.016	0.031	0.145
.7	0.000	0.000	0.001	0.001	0.001	0.002	0.004	0.022	0.044	0.195
1	0.001	0.001	0.001	0.001	0.002	0.003	0.006	0.031	0.062	0.265
10	0.006	0.007	0.008	0.012	0.017	0.033	0.063	0.265	0.456	0.967
25	0.016	0.017	0.021	0.030	0.043	0.081	0.148	0.532	0.781	1.000
100	0.059	0.065	0.078	0.115	0.162	0.271	0.457	0.966	1.000	1.000
1000	0.388	0.422	0.482	0.625	0.758	0.951	1.000	1.000	1.000	1.000

Thus, if the optimal allocation is not obeyed, and maximum quota sampling blindly applied, the posterior standard deviation of $\tilde{\mu}_u$ will be 0.312 in spite of the seemingly high prior correlation, $\rho_{ub} = 0.95$. Positive intra-PSU correlation, i.e., $\rho_c>0$, will increase the relative efficiency of total quota sampling; but does not reduce the posterior standard deviation. For example, with $\rho_c = .5$, for the $c_1 = 0.1$, $\rho_{ub} = 0.95$ combination with optimum n_u,

$$V''^{\frac{1}{2}}_{uu} = 0.0326,$$

whereas if full quota sampling were employed the efficiency would be 0.0108 with

$$V''^{\frac{1}{2}}_{uu} = 0.313,$$

a negligible change from the case of $\rho_c=0$.

In Tables 2 and 3, consider an example where some quota sampling is desirable, as with $c_1= 10$ and $\rho_{ub}= 0.999$. Table 3 shows that for that combination (with $\rho_c = 0$) total quota sampling has an efficiency of 0.967. But the main idea in quota sampling is to save money, hence we might ask, "What if the budget q^* were cut in half, making possible only 8 full probability or 80 quota observations per PSU?" Comparison of variances shows that with total quota sampling at half the budget, the precision will be 0.782 times the optimum at full budget. If, however, the new reduced budget were to be totally devoted to full probability sampling, the efficiency would be only 0.205. Since 8000 observations is a very large sample, we might consider working with one-eighth of the original budget or $q^* = 200$, yielding 20 quota observations per PSU. In that case the efficiency drops to 0.365, but the posterior standard deviation of $\tilde{\mu}_u$ is still as small as 0.083.

c. $\tilde{\mu}_u$ and $\tilde{\beta}$ correlated, $\rho_c > 0$.

In the more general situation of $|\rho_{u\beta}| > 0$, (i.e., $V'_{uu} \neq V'_{ub}$), and $\rho_c > 0$, we must use formulas (4.16) and (4.17) for n_u, and the simplicity and invariance of the previous case are lost. In the full report of this work we examine several examples that illustrate this case, but restrictions of space do not permit discussion here.

CONCLUSION

The examples in this paper, as well as many others examined by the author by means of the time-sharing computer, indicate that the optimal use of quota sampling, even with a small number of PSUs, requires very high prior correlations between $\tilde{\mu}_u$ and $\tilde{\mu}_b$ and large variances of the measurement processes. Relative cost appears to have little impact on the results. It should come as no great surprise that in order to justify quota sampling, one ought to believe the process means to be correlated, but just how high a correlation is necessary may be of some interest.

Two final remarks:

1. First, we have discussed a model in which one of the measurement processes is unbiased, i.e. μ_u can be estimated without systematic error. In the light of increasing difficulties in eliciting response in household surveys, even under the best conditions, one wonders how many survey researchers would accept the realism of the model.

2. Secondly, our model has not really encompassed the case that leads to certain uses of quota sampling, especially in the surveying of current public opinion. Here quota methods are employed because there simply is not enough calendar time to get high response through follow-ups. In terms of our model, k_u is infinite (or at least greater than $k^*/_p$). Political pollsters are able to demonstrate the accuracy of their estimates by comparison with actual election returns, and their records are impressive. Can sociologists, urban planners, and market researchers who do not face equally severe timetables be sure of the magnitudes and directions of the biases in their use of quota sampling?

Bayesian methods are valuable, not because anyone seriously believes that a prior distribution is easy to assess, nor that it is psychologically stationary,but rather because these methods help the decision maker to better understand the implications of actions that he might otherwise choose out of habit, convenience, or questionable advice.